PHIẾU BÀI TẬP VỀ CĂN THỨC BẬC HAI - SỐ 2

Bài 1: Thực hiện phép tính

$A=(\sqrt{75}+\sqrt{243}-\sqrt{48}):\sqrt{3}$

$B=(20\sqrt{12}-15\sqrt{27}):5\sqrt{3}$

$C=(\sqrt{2}+1)^2+(\sqrt{2}-1)^2$

$D=(\sqrt{28}-2\sqrt{3}+\sqrt{7})\sqrt{7}+\sqrt{84}$

Giải:

$A=(\sqrt{75}+\sqrt{243}-\sqrt{48}):\sqrt{3}$

$\dfrac{\sqrt{A}}{\sqrt{B}}=\sqrt{\dfrac{A}{B}} \quad (A\geq 0; B>0)$

$=\sqrt{75}:\sqrt{3}+\sqrt{243}:\sqrt{3}-\sqrt{48}:\sqrt{3}$

$=\sqrt{\dfrac{75}{3}}+\sqrt{\dfrac{243}{3}}-\sqrt{\dfrac{48}{3}}$

$=\sqrt{25}+\sqrt{81}-\sqrt{16}$

$=5+9-4$

$=10$

$B=(20\sqrt{12}-15\sqrt{27}):5\sqrt{3}$

$=20\sqrt{12}:5\sqrt{3}-15\sqrt{27}:5\sqrt{3}$

$=4.\sqrt{\dfrac{12}{3}}-3.\sqrt{\dfrac{27}{3}}$

$=4.\sqrt{4}-3\sqrt{9}$

$=4.2-3.3$

$=-1$

$C=(\sqrt{2}+1)^2+(\sqrt{2}-1)^2$

$=\sqrt{2}^2+2.\sqrt{2}.1+1^2+\sqrt{2}^2-2.\sqrt{2}.1+1^2$

$=2+2\sqrt{2}+1+2-2\sqrt{2}+1$

$=6$

$D=(\sqrt{28}-2\sqrt{3}+\sqrt{7})\sqrt{7}+\sqrt{84}$

$\sqrt{A}.\sqrt{B}=\sqrt{AB} \quad (A,B\geq 0)$

$=\sqrt{28}.\sqrt{7}-2\sqrt{3}.\sqrt{7}+\sqrt{7}^2+\sqrt{84}$

$=\sqrt{196}-2\sqrt{21}+7+\sqrt{4.21}$

$=14-2\sqrt{21}+7+2\sqrt{21}$

$=21$

Bài 2: Trục căn thức ở mẫu

a) $\dfrac{3}{\sqrt{5}}$

b) $\dfrac{2\sqrt{3}}{\sqrt{2}}$

c) $\dfrac{2+\sqrt{3}}{2-\sqrt{3}}$

d) $\dfrac{1}{\sqrt{2}+\sqrt{3}}$

e) $\dfrac{\sqrt{2}+1}{\sqrt{2}-1}$

f) $\dfrac{3\sqrt{2}}{\sqrt{3}+1}$

Giải:

a) $\frac{3}{\sqrt{5}}=\frac{3.\sqrt{5}}{\sqrt{5}.\sqrt{5}}=\frac{3.\sqrt{5}}{\sqrt{5}^2}=\frac{3.\sqrt{5}}{5}$

b) $\frac{2\sqrt{3}}{\sqrt{2}}=\frac{2\sqrt{3}.\sqrt{2}}{\sqrt{2}.\sqrt{2}}=\frac{2\sqrt{3.2}}{\sqrt{2}^2}=\frac{2\sqrt{6}}{2}=\sqrt{6}$

d) $\frac{1}{\sqrt{2}+\sqrt{3}}=\frac{\sqrt{2}-\sqrt{3}}{(\sqrt{2}+\sqrt{3})(\sqrt{2}-\sqrt{3})}$

$=\frac{\sqrt{2}-\sqrt{3}}{\sqrt{2}^2-\sqrt{3}^2}=\frac{\sqrt{2}-\sqrt{3}}{2-3}=\sqrt{3}-\sqrt{2}$

c) $\frac{2+\sqrt{3}}{2-\sqrt{3}}=\frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}$

$=\frac{(2+\sqrt{3})^2}{2^2-\sqrt{3}^2}=\frac{2^2+2.2.\sqrt{3}+\sqrt{3}^2}{4-3}=7+4\sqrt{3}$

$(a+b)(a-b)=a^2-b^2$

e) $\frac{\sqrt{2}+1}{\sqrt{2}-1}=\frac{(\sqrt{2}+1)(\sqrt{2}+1)}{(\sqrt{2}-1)(\sqrt{2}+1)}=\frac{(\sqrt{2}+1)^2}{\sqrt{2}^2-1}$

$=\frac{\sqrt{2}^2+2.\sqrt{2}.1+1}{2-1}=3+2\sqrt{2}$

f) $\frac{3\sqrt{2}}{\sqrt{3}+1}=\frac{3\sqrt{2}(\sqrt{3}-1)}{(\sqrt{3}+1)(\sqrt{3}-1)}$

$=\frac{3\sqrt{2.3}-1.3\sqrt{2}}{\sqrt{3}^2-1}=\frac{3\sqrt{6}-3\sqrt{2}}{2}$

Bài 3: Rút gọn biểu thức

$A=\frac{10+2\sqrt{10}}{\sqrt{5}+\sqrt{2}}+\frac{8}{1-\sqrt{5}}$

$B=\frac{2\sqrt{8}-\sqrt{12}}{\sqrt{18}-\sqrt{48}}-\frac{\sqrt{5}+\sqrt{27}}{\sqrt{30}+\sqrt{162}}$

$C=2\sqrt{\frac{16}{3}}-3\sqrt{\frac{1}{27}}-6\sqrt{\frac{4}{75}}$

$D=\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}+\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$

Giải:

$$A=\frac{10+2\sqrt{10}}{\sqrt{5}+\sqrt{2}}+\frac{8}{1-\sqrt{5}}$$
$$=\frac{\sqrt{10}.\sqrt{10}+2\sqrt{10}}{\sqrt{5}+\sqrt{2}}+\frac{8}{1-\sqrt{5}}$$
$$=\frac{\sqrt{10}.\sqrt{2.5}+\sqrt{2}.\sqrt{2}.\sqrt{10}}{\sqrt{5}+\sqrt{2}}+\frac{8}{1-\sqrt{5}}$$
$$=\frac{\sqrt{10}.\sqrt{2}.(\sqrt{5}+\sqrt{2})}{\sqrt{5}+\sqrt{2}}+\frac{8}{1-\sqrt{5}}$$
$$=2\sqrt{5}+\frac{8}{1-\sqrt{5}}=\frac{2\sqrt{5}(1-\sqrt{5})+8}{1-\sqrt{5}}$$
$$=\frac{2\sqrt{5}-2.\sqrt{5}^2+8}{1-\sqrt{5}}$$
$$=\frac{2\sqrt{5}-2}{1-\sqrt{5}}=\frac{-2(1-\sqrt{5})}{1-\sqrt{5}}=-2$$

$$D=\sqrt{\frac{2-\sqrt{3}}{2+\sqrt{3}}}+\sqrt{\frac{2+\sqrt{3}}{2-\sqrt{3}}}$$
$$=\sqrt{\frac{(2-\sqrt{3})(2-\sqrt{3})}{(2+\sqrt{3})(2-\sqrt{3})}}+\sqrt{\frac{(2+\sqrt{3})(2+\sqrt{3})}{(2-\sqrt{3})(2+\sqrt{3})}}$$
$$=\sqrt{\frac{(2-\sqrt{3})^2}{2^2-\sqrt{3}^2}}+\sqrt{\frac{(2+\sqrt{3})^2}{2^2-\sqrt{3}^2}}$$
$$=\frac{\sqrt{(2-\sqrt{3})^2}}{\sqrt{2^2-\sqrt{3}^2}}+\frac{\sqrt{(2+\sqrt{3})^2}}{\sqrt{2^2-\sqrt{3}^2}}$$
$$=\frac{\left|2-\sqrt{3}\right|}{1}+\frac{\left|2+\sqrt{3}\right|}{1}$$
$$=2-\sqrt{3}+2+\sqrt{3}=4$$

vi $2>\sqrt{3}$

$$B=\frac{2\sqrt{8}-\sqrt{12}}{\sqrt{18}-\sqrt{48}}-\frac{\sqrt{5}+\sqrt{27}}{\sqrt{30}+\sqrt{162}}$$
$$=\frac{2\sqrt{4.2}-\sqrt{4.3}}{\sqrt{9.2}-\sqrt{16.3}}-\frac{\sqrt{5}+\sqrt{27}}{\sqrt{6.5}+\sqrt{6.27}}$$
$$=\frac{4\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-4\sqrt{3}}-\frac{\sqrt{5}+\sqrt{27}}{\sqrt{6}.\sqrt{5}+\sqrt{6}.\sqrt{27}}$$
$$=\frac{4\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-4\sqrt{3}}-\frac{\sqrt{5}+\sqrt{27}}{\sqrt{6}.(\sqrt{5}+\sqrt{27})}$$
$$=\frac{4\sqrt{2}-2\sqrt{3}}{3\sqrt{2}-4\sqrt{3}}-\frac{1}{\sqrt{6}}$$
$$=\frac{\sqrt{6}(4\sqrt{2}-2\sqrt{3})-(3\sqrt{2}-4\sqrt{3})}{\sqrt{6}(3\sqrt{2}-4\sqrt{3})}$$
$$=\frac{4\sqrt{12}-2\sqrt{18}-3\sqrt{2}+4\sqrt{3}}{\sqrt{6}(3\sqrt{2}-4\sqrt{3})}$$
$$=\frac{8\sqrt{3}-6\sqrt{2}-3\sqrt{2}+4\sqrt{3}}{\sqrt{6}(3\sqrt{2}-4\sqrt{3})}$$
$$=\frac{12\sqrt{3}-9\sqrt{2}}{\sqrt{6}(3\sqrt{2}-4\sqrt{3})}=\frac{-3(3\sqrt{2}-4\sqrt{3})}{\sqrt{6}(3\sqrt{2}-4\sqrt{3})}=\frac{-3}{\sqrt{6}}=\frac{-\sqrt{6}}{2}$$

$$C=2\sqrt{\frac{16}{3}}-3\sqrt{\frac{1}{27}}-6\sqrt{\frac{4}{75}}$$
$$=2.\frac{\sqrt{16}}{\sqrt{3}}-3.\frac{1}{\sqrt{27}}-6.\frac{\sqrt{4}}{\sqrt{75}}$$
$$=2.\frac{4}{\sqrt{3}}-3.\frac{1}{3\sqrt{3}}-6.\frac{2}{5\sqrt{3}}$$
$$=\frac{8}{\sqrt{3}}-\frac{1}{\sqrt{3}}-\frac{12}{5\sqrt{3}}$$
$$=\frac{23}{5\sqrt{3}}=\frac{23\sqrt{3}}{15}$$

Bài 4: Rút gọn biểu thức

$A=\dfrac{\sqrt{3-\sqrt{5}}.(3+\sqrt{5})}{\sqrt{10}+\sqrt{2}}$

$B=\dfrac{4}{\sqrt{3}+1}+\dfrac{1}{\sqrt{3}-2}+\dfrac{6}{\sqrt{3}-3}$

$C=\sqrt{4-\sqrt{9+4\sqrt{2}}}$

$D=\dfrac{1}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\dfrac{1}{\sqrt{2}-\sqrt{2-\sqrt{3}}}$

Giải:

$A=\dfrac{\sqrt{3-\sqrt{5}}.(3+\sqrt{5})}{\sqrt{10}+\sqrt{2}}$

$\sqrt{2}.A=\dfrac{\sqrt{2}.\sqrt{3-\sqrt{5}}.(3+\sqrt{5})}{\sqrt{10}+\sqrt{2}}$

$\sqrt{2}.A=\dfrac{\sqrt{6-2\sqrt{5}}.(3+\sqrt{5})}{\sqrt{10}+\sqrt{2}}$

$\sqrt{2}.A=\dfrac{\sqrt{5-2\sqrt{5}+1}.(3+\sqrt{5})}{\sqrt{10}+\sqrt{2}}$

$\sqrt{2}.A=\dfrac{\sqrt{(\sqrt{5}-1)^2}.(3+\sqrt{5})}{\sqrt{2}.(\sqrt{5}+1)}$

$\sqrt{2}.A=\dfrac{\left|\sqrt{5}-1\right|.(3+\sqrt{5})}{\sqrt{2}.(\sqrt{5}+1)}$

$\sqrt{2}.A=\dfrac{(\sqrt{5}-1).(3+\sqrt{5})}{\sqrt{2}.(\sqrt{5}+1)}$

$A=\dfrac{(\sqrt{5}-1).(3+\sqrt{5})}{2.(\sqrt{5}+1)}=\dfrac{3\sqrt{5}+5-3-\sqrt{5}}{2.(\sqrt{5}+1)}$

$=\dfrac{2\sqrt{5}+2}{2.(\sqrt{5}+1)}=\dfrac{2.(\sqrt{5}+1)}{2.(\sqrt{5}+1)}=1$

$B=\dfrac{4}{\sqrt{3}+1}+\dfrac{1}{\sqrt{3}-2}+\dfrac{6}{\sqrt{3}-3}$

$=\dfrac{4(\sqrt{3}+1)}{(\sqrt{3}+1)(\sqrt{3}-1)}+\dfrac{\sqrt{3}+2}{(\sqrt{3}-2)(\sqrt{3}+2)}+\dfrac{6(\sqrt{3}+3)}{(\sqrt{3}-3)(\sqrt{3}+3)}$

$=\dfrac{4\sqrt{3}+4}{\sqrt{3}^2-1}+\dfrac{\sqrt{3}+2}{\sqrt{3}^2-2^2}+\dfrac{6\sqrt{3}+18}{\sqrt{3}^2-3^2}$

$=\dfrac{4\sqrt{3}+4}{2}+\dfrac{\sqrt{3}+2}{-1}+\dfrac{6(\sqrt{3}+3)}{-6}$

$=2\sqrt{3}+2-\sqrt{3}-2-\sqrt{3}-3=-3$

$C=\sqrt{4-\sqrt{9+4\sqrt{2}}}$

$=\sqrt{4-\sqrt{8+2.2\sqrt{2}.1+1}}$

$=\sqrt{4-\sqrt{(2\sqrt{2})^2+2.2\sqrt{2}.1+1^2}}$

$=\sqrt{4-\sqrt{(2\sqrt{2}+1)^2}}$

$=\sqrt{4-\left|2\sqrt{2}+1\right|}$ *vì* $2\sqrt{2}+1>0$

$=\sqrt{4-2\sqrt{2}-1}$

$=\sqrt{2-2.\sqrt{2}.1+1}$

$=\sqrt{(\sqrt{2}-1)^2}=\left|\sqrt{2}-1\right|=\sqrt{2}-1$ *vì* $\sqrt{2}>1$

$$D=\frac{1}{\sqrt{2}+\sqrt{2+\sqrt{3}}}+\frac{1}{\sqrt{2}-\sqrt{2-\sqrt{3}}}$$
$$=\frac{\sqrt{2}}{\sqrt{2}^2+\sqrt{4+2\sqrt{3}}}+\frac{\sqrt{2}}{\sqrt{2}^2-\sqrt{4-2\sqrt{3}}}$$
$$=\frac{\sqrt{2}}{2+\sqrt{3+2\sqrt{3}+1}}+\frac{\sqrt{2}}{2-\sqrt{3-2\sqrt{3}+1}}$$
$$=\frac{\sqrt{2}}{2+\sqrt{(\sqrt{3}+1)^2}}+\frac{\sqrt{2}}{2+\sqrt{(\sqrt{3}-1)^2}}$$
$$=\frac{\sqrt{2}}{2+\left|\sqrt{3}+1\right|}+\frac{\sqrt{2}}{2-\left|\sqrt{3}-1\right|}$$
$$=\frac{\sqrt{2}}{2+\sqrt{3}+1}+\frac{\sqrt{2}}{2-(\sqrt{3}-1)}$$
$$=\frac{\sqrt{2}}{3+\sqrt{3}}+\frac{\sqrt{2}}{3-\sqrt{3}}=\frac{\sqrt{2}(3-\sqrt{3})+\sqrt{2}(3+\sqrt{3})}{(3+\sqrt{3})(3-\sqrt{3})}$$
$$=\frac{3\sqrt{2}-\sqrt{6}+3\sqrt{2}+\sqrt{6})}{3^2-\sqrt{3}^2}$$
$$=\sqrt{2}$$

Câu 5: Rút gọn biểu thức

$A=\sqrt{3-\sqrt{5}}+\sqrt{3+\sqrt{5}}$

$B=(\sqrt{2}+1)^3-(\sqrt{2}-1)^3$

$C=\left(1-\frac{5+\sqrt{5}}{1+\sqrt{5}}\right)\left(\frac{5-\sqrt{5}}{1-\sqrt{5}}-1\right)$

$D=\sqrt{4+\sqrt{10+2\sqrt{5}}}+\sqrt{4-\sqrt{10+2\sqrt{5}}}$

Giải:

$A=\sqrt{3-\sqrt{5}}+\sqrt{3+\sqrt{5}}$

$\sqrt{2}A=\sqrt{6-2\sqrt{5}}+\sqrt{6+2\sqrt{5}}$

$\sqrt{2}A=\sqrt{5-2\sqrt{5}+1}+\sqrt{5+2\sqrt{5}+1}$

$\sqrt{2}A=\sqrt{\sqrt{5}^2-2.\sqrt{5}.1+1^2}+\sqrt{\sqrt{5}^2+2.\sqrt{5}.1+1^2}$

$\sqrt{2}A=\sqrt{(\sqrt{5}-1)^2}+\sqrt{(\sqrt{5}+1)^2}$

$\sqrt{2}A=\left|\sqrt{5}-1\right|+\left|\sqrt{5}+1\right|$

$\sqrt{2}A=\sqrt{5}-1+\sqrt{5}+1 \quad vi\ \sqrt{5}>1$

$\sqrt{2}A=2\sqrt{5}$

$A=\sqrt{2}.\sqrt{5}=\sqrt{10}$

$C=\left(1-\dfrac{5+\sqrt{5}}{1+\sqrt{5}}\right)\left(\dfrac{5-\sqrt{5}}{1-\sqrt{5}}-1\right)$

$=\left(1-\dfrac{\sqrt{5}(\sqrt{5}+1)}{1+\sqrt{5}}\right)\left(\dfrac{\sqrt{5}(\sqrt{5}-1)}{1-\sqrt{5}}-1\right)$

$=\left(1-\sqrt{5}\right)\left(-\sqrt{5}-1\right)$

$=\left(\sqrt{5}-1\right)\left(\sqrt{5}+1\right)$

$=\sqrt{5}^2-1=4$

$B=(\sqrt{2}+1)^3-(\sqrt{2}-1)^3$

$=(\sqrt{2}^3+3.\sqrt{2}^2.1+3.\sqrt{2}.1^2+1^3)-(\sqrt{2}^3-3.\sqrt{2}^2.1+3.\sqrt{2}.1^2-1^3)$

$=(2\sqrt{2}+6+3\sqrt{2}+1)-(2\sqrt{2}-6+3\sqrt{2}-1)$

$=14$

$D=\sqrt{4+\sqrt{10+2\sqrt{5}}}+\sqrt{4-\sqrt{10+2\sqrt{5}}}$

$D^2=(\sqrt{4+\sqrt{10+2\sqrt{5}}}+\sqrt{4-\sqrt{10+2\sqrt{5}}})^2$

$D^2=4+\sqrt{10+2\sqrt{5}}+2.\sqrt{4+\sqrt{10+2\sqrt{5}}}+...$

$\sqrt{4-\sqrt{10+2\sqrt{5}}}+4-\sqrt{10+2\sqrt{5}}$

$D^2=8+2.\sqrt{4^2-\sqrt{10+2\sqrt{5}}^2}$

$D^2=8+2.\sqrt{16-(10+2\sqrt{5})}$

$D^2=8+2.\sqrt{6-2\sqrt{5}}$

$D^2=8+2.\sqrt{\sqrt{5}^2-2\sqrt{5}+1}$

$D^2=8+2.\sqrt{(\sqrt{5}-1)^2}$

$D^2=8+2.\left|\sqrt{5}-1\right|=8+2(\sqrt{5}-1)=6+2\sqrt{5}$

$D=\sqrt{6+2\sqrt{5}}=\sqrt{\sqrt{5}^2+2\sqrt{5}+1}=\sqrt{(\sqrt{5}+1)^2}$

$vi\ D>0$

$=\left|\sqrt{5}+1\right|=\sqrt{5}+1 \qquad vi\ \sqrt{5}+1>0$

Bài 6: Tìm x biết

a) $\sqrt{x-1}+\sqrt{4x-4}-\sqrt{25x-25}+2=0$

b) $\sqrt{16x+16}-\sqrt{9x+9}+\sqrt{4x+4}+\sqrt{x+1}=16$

c) $\sqrt{4x+20}+\sqrt{x+5}-\dfrac{1}{3}\sqrt{9x+45}=4$

d) $\dfrac{1}{3}\sqrt{2x}-\sqrt{8x}+\sqrt{18x}-10=2$

Giải:

$a)\sqrt{x-1}+\sqrt{4x-4}-\sqrt{25x-25}+2=0 \qquad DK: x\geq 1$

$\Leftrightarrow \sqrt{x-1}+\sqrt{4(x-1)}-\sqrt{25(x-1)}+2=0$

$\Leftrightarrow \sqrt{x-1}+2\sqrt{x-1}-5\sqrt{x-1}+2=0$

$\Leftrightarrow -2\sqrt{x-1}=-2$

$\Leftrightarrow \sqrt{x-1}=1$

$\Leftrightarrow \sqrt{x-1}^2=1^2$

$\Leftrightarrow x-1=1$

$\Leftrightarrow x=2 \qquad (tm)$

$b)\sqrt{16x+16}-\sqrt{9x+9}+\sqrt{4x+4}+\sqrt{x+1}=16 \qquad DK: x\geq -1$

$\Leftrightarrow \sqrt{16(x+1)}-\sqrt{9(x+1)}+\sqrt{4(x+1)}+\sqrt{x+1}=16$

$\Leftrightarrow 4\sqrt{x+1}-3\sqrt{x+1}+2\sqrt{x+1}+\sqrt{x+1}=16$

$\Leftrightarrow 4\sqrt{x+1}=16$

$\Leftrightarrow \sqrt{x+1}=4$

$\Leftrightarrow \sqrt{x+1}^2=4^2$

$\Leftrightarrow x+1=16$

$\Leftrightarrow x=15 \qquad (tm)$

$c)\sqrt{4x+20}+\sqrt{x+5}-\frac{1}{3}\sqrt{9x+45}=4 \quad DK: x\geq -5$

$\Leftrightarrow \sqrt{4(x+5)}+\sqrt{x+5}-\frac{1}{3}\sqrt{9(x+5)}=4$

$\Leftrightarrow 2\sqrt{x+5}+\sqrt{x+5}-\frac{1}{3}.3.\sqrt{x+5}=4$

$\Leftrightarrow 2.\sqrt{x+5}=4$

$\Leftrightarrow \sqrt{x+5}=2$

$\Leftrightarrow \sqrt{x+5}^2=2^2$

$\Leftrightarrow x+5=4$

$\Leftrightarrow x=-1 \quad (tm)$

$d)\ \frac{1}{3}\sqrt{2x}-\sqrt{8x}+\sqrt{18x}-10=2 \qquad DK: x\geq 0$

$\Leftrightarrow \frac{1}{3}\sqrt{2x}-\sqrt{4.2x}+\sqrt{9.2x}-10=2$

$\Leftrightarrow \frac{1}{3}\sqrt{2x}-2\sqrt{2x}+3\sqrt{2x}-10=2$

$\Leftrightarrow \frac{4}{3}\sqrt{2x}=12$

$\Leftrightarrow \sqrt{2x}=9$

$\Leftrightarrow \sqrt{2x}^2=9^2$

$\Leftrightarrow 2x=81$

$\Leftrightarrow x=\frac{81}{2} \qquad (tm)$